

Marquette University

**e-Publications@Marquette**

---

Electrical and Computer Engineering Faculty  
Research and Publications

Electrical and Computer Engineering,  
Department of

---

2-2019

## Permanent Magnet Vernier Machine: A Review

Fan Wu

Ayman M. EL-Refaie

Follow this and additional works at: [https://epublications.marquette.edu/electric\\_fac](https://epublications.marquette.edu/electric_fac)



Part of the [Computer Engineering Commons](#), and the [Electrical and Computer Engineering Commons](#)

---

Marquette University

**e-Publications@Marquette**

***Electrical and Computer Engineering Faculty Research and Publications/College of Engineering***

***This paper is NOT THE PUBLISHED VERSION; but the author's final, peer-reviewed manuscript.*** The published version may be accessed by following the link in the citation below.

*IET Electric Power Applications*, Vol. 13, No. 2 (February 2019): 127 - 137. [DOI](#). This article is © IEEE and permission has been granted for this version to appear in [e-Publications@Marquette](#). IEEE does not grant permission for this article to be further copied/distributed or hosted elsewhere without the express permission from IEEE.

# Permanent Magnet Vernier Machine: A Review

Fan Wu

Marquette University, Milwaukee, WI, US

Ayman M. El-Refaie

Marquette University, Milwaukee, WI, US

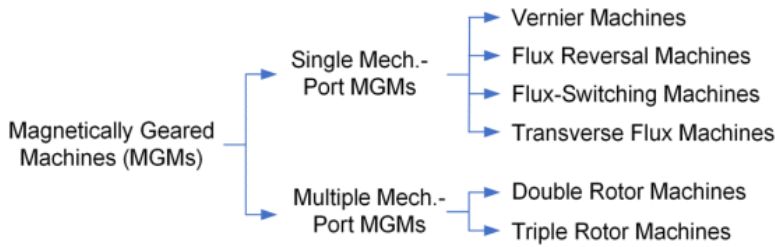
## Abstract:

Permanent magnet vernier machines (PMVMs) gained a lot of interest over the past couple of decades. This is mainly due to their high torque density enabled by the magnetic gearing effect. This study will provide a thorough review of recent advances in PMVMs. This review will cover the principle of operation and nature of magnetic gearing in PMVMs, and a better understanding of novel PMVM topologies using different winding configuration as well as different modulation poles and rotor structures. Detailed discussions on the choice of gear ratio, slot-pole combinations, design optimisation and role of advanced materials in PMVMs will be presented. This will provide an update on the current state-of-the art as well as future areas of research. Furthermore, the power factor issue, fault

tolerance as well as cost reduction will be discussed highlighting the gap between the current state-of-the-art and what is needed in practical applications.

## SECTION 1. Introduction

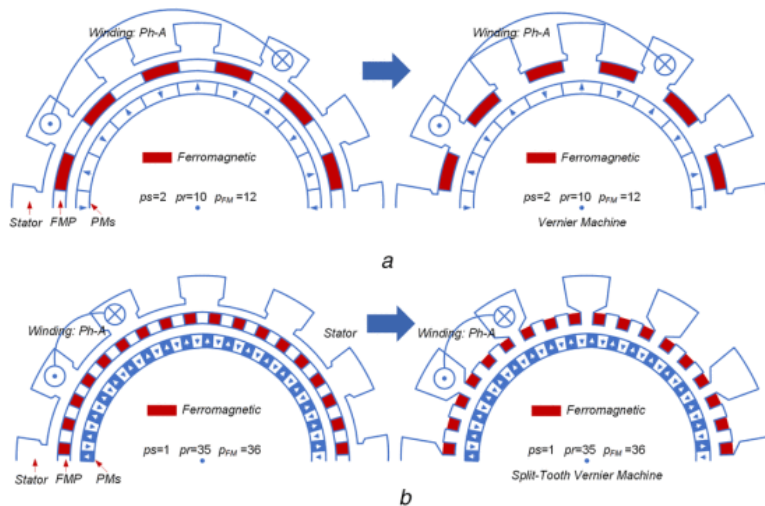
Vernier machine (VM), flux-reversal machine, flux switching machine as well as the transverse flux machine can be regarded as a synchronous machine integrated with a magnetic gear [1–3]. All those machine topologies can be grouped into a unified classification of magnetically geared machine (MGM) family, as given in Fig. 1.



**Fig. 1.** Classification of magnetically geared machines

The classification of the MGM is based on the number of mechanical ports. The four electrical machines mentioned above belong to the sub-category of single mechanical-port MGMs in which only one of the three parts of an MGM ((i) coil-wound core, (ii) flux-modulation-pole (FMP) and (iii) permanent magnet (PM) rotor), is able to spin. The other sub-category includes multiple mechanical-port MGMs. A good example of this category is the double-rotor machine used in power split systems such as the continuous variable transmission devices [4].

Fig. 2 shows a simplified evolution from MGMs with a fixed FMP to VMs with either split-tooth or non-split tooth. The inherent relationship of the three variables, i.e. no. of stator winding MMF pole pairs ( $p_s$ ), no. of rotor pole pairs ( $p_r$ ), and no. of FMPs ( $p_{FM}$ ), should meet a constraint of  $p_r = p_{FM} \pm p_s$ .



**Fig. 2.** PMVMs derived from fixed FMP magnetically geared dual mechanical port machines  
(a) Non-split-tooth, distributed winding (DW), (b) Split-tooth, FSCW

The first VM dates back to 1960 when it was referred to as a reluctance-type inductor synchronous motor [5]. Interest in VMs has been revived with advancements in PMs and PM brush-less machines

[6]. Several publications focused on providing a generic/unified torque production analysis for the surface PMVM (SPMVM). This helped to provide a better understanding of the principle of operation as well as laying a solid foundation for designing permanent-magnet vernier machines (PMVMs) [7, 8].

In regular electric drives, mechanical gears address the speed/torque mismatch between the high-speed motor shaft and the low-speed prime-mover/process shaft. Gear lead to high maintenance cost as well as lifetime challenges because its lifetime is typically shorter than electrical machines. On the other hand, low-speed high-torque direct-drive systems result in physically large machines. PMVMs, some topologies of which have torque producing capability that exceeds that of a conventional electrical machine by 2–3 times, can be a good solution for this well-known trade-off.

Due to the significant low-speed high-torque advantages, VMs are undoubtedly a key potential enabler for direct-drive applications. Typically, torque ripple of a VM is fairly small, which is also a desirable feature that reduces acoustic noise and vibration. Based on all these perceived advantages, VMs are being considered as potential promising alternatives to conventional electrical machines for a wide range of applications [9–13]. Such applications include

- *Wave energy extraction*: either rotary or linear machine,  $\sim 0.5$  m/s and  $>1$  MW.
- *Wind turbine*: direct-drive generator, speed: 15–30 r/min, 10–100 kW (small), and 2–5 MW (large).
- *In-wheel motor*: outer-rotor machine,  $\sim 75$  kW (consumer electric vehicles), rated/max. speed: 500/1800 r/min.
- *Auto-focusing lens*: axial-flux motor,  $<100$  r/min and 1–2 W
- *Free-piston generator*: linear machine, 10–20 m/s and  $\sim 10$  kW.

Despite the fact that PMVM is superior at slow speeds in terms of torque performance and efficiency, a comparative study between an optimised nine-stator-slot/27-FMP/48-rotor-pole PMVM and the 2010 Toyota Prius IPM motor shows that the PMVM might have limitations for adjustable-speed applications due to extremely narrow constant power speed range (CPSR) as well as lower efficiency within that range [14]. However, another recent case study demonstrated that an outer-rotor PMVM is quite appealing in terms of wide CPSR in contrast to its fractional slot concentrated winding (FSCW)-surface PM (SPM) counterparts [15]. Those conflicting results raise concerns about previous attempts to propose this type of machine for in-wheel motors [16, 17].

Furthermore, this type of machine also suffers from fairly low-power factor compared with regular PM synchronous machines [18, 19]. This undesired feature is identified as a result of excessive harmonic leakage flux in the air gap. To maintain a reasonable inverter capacity, this issue should be carefully addressed.

Even though PMVMs have been receiving the major attention, non-magnet VMs are of great interest to either low-cost or safety-critical applications. An example is a stator DC winding excited vernier reluctance machine that takes advantages of its doubly salient structure as well as stator DC field windings [20–24].

As a summary, the on-going research activities related to VMs are focusing on three primary areas:

- *High torque density*: pursue novel topologies to achieve higher torque-density.
- *Higher power factor*: (i) reduce leakage flux path through better-designed magnetic circuits and/or advanced materials; (ii) utilise MMF harmonics as useful torque-producing components.
- *Lower cost*: Reduce/eliminate rare-earth materials through optimised novel topologies and advanced materials.

This study presents a survey of PMVMs. The focus will be on the research activities that took place over the past 5–10 years. The paper is organised as follows: Section 2 covers efforts addressing torque production in PMVMs. Section 3 provides an analysis of torque production in PMVMs based on winding function approaches. Section 4 covers the design aspects of the three major components of a VM, i.e. winding, FMP as well as a rotor, while Section 5 covers a variety of PMVM novel topologies and their evolution. Section 6 covers design issues and optimisation. Section 7 covers the low-power factor issue and highlights the available solutions. Section 8 discusses the use of advanced materials and their potential impact on VMs. Reliability-related issues are covered in Section 9 followed by conclusions.

## SECTION 2. Torque production

There have been lots of research efforts dedicated to the understanding of torque producing principle in PMVMs. Analytical modelling of VMs generally starts with the influence of air gap permeance on winding MMF and PM field, establishing a mathematic description of the so-called ‘flux modulation’ or ‘magnetic gearing effect’ [25].

The magnetic gearing effect is essentially the flux modulation effect of the specific permeance ( $P$ ) as a function of stator mechanical angle ( $\theta$ ) introduced by stator slotting on PM-generated MMF ( $F_{PM}$ ). For simplicity, if only the first-order component of  $P$  and  $F_{PM}$  are considered, the air gap flux density can be derived as

$$\begin{aligned}
 B_{PM} &= P(\theta)F_{PM}(\theta, \theta_m) \\
 &\simeq [P_0 - P_1 \cos(p_{FM}\theta)][F_{PM1} \cos(p_r(\theta - \theta_m))] \\
 &= P_0 F_{PM1} \cos(p_r(\theta - \theta_m)) - \frac{1}{2} P_1 F_{PM1} \\
 &\quad \times [\cos((p_{FM} - p_r)\theta + p_r \theta_m) + \cos((p_{FM} + p_r)\theta - p_r \theta_m)],
 \end{aligned} \tag{1}$$

where  $\theta$  is the stator mech. angle;  $\theta_m = \omega t$  is the rotor mech. angle;  $p_s$  and  $p_r$  are pole pair of stator and rotor MMFs, respectively;  $p_{FM}$  is the number of FMPs which is exactly the number of slots or split slots. The second term of (1) indicates that both  $(p_{FM} \pm p_r)$ -pole-pair flux density components can be obtained as a result of the flux modulation effect.

According to the generic torque equation derived by Toba and Lipo [7], the average torque can be written as follows:

$$T = \frac{3\sqrt{2}}{\pi} \Psi l_{ef} p_r k_{d1} k_{p1} N_s I (P_0 F_{PM1} \mp \frac{P_1 F_{PM1}}{2} \frac{p_r}{p_s}), \tag{2}$$

where ‘-’ is applied when  $p_r = p_{FM} - p_s$ ; ‘+’ is applied when  $p_r = p_{FM} + p_s$ ;  $\Psi$  is the flux linkage;  $l_{ef}$  is the effective axial length of the machine;  $k_{d1}$  and  $k_{p1}$  are winding distributed and pitch factor, respectively;  $N_s$  is the series turns per phase;  $I$  is the stator winding current.

As can be seen from (2), the choice of  $p_r = p_{FM} - p_s$  leads to higher torque production. The so-called ‘harmonic coupling’, which refers to the interaction between the harmonic component of the PM field and the slot harmonic component of the coil MMF, is the reason why VMs enjoy a higher torque density. Specifically, there are two torque components in a PMVM rather than only one component in a conventional permanent magnet synchronous machine (PMSM): (i) the first one is the synchronous reaction torque produced by PMs as well as stator MMF, of which the electrical angular speed is defined by the pole pitch of the stator winding; (ii) the second one is the reluctance torque produced by the PMs and the fundamental slot harmonic component of MMF which rotates at a higher synchronous speed [26].

A general instantaneous torque equation for PMVMs has been developed to analyse average and ripple torque [27, 28]. This enables quantitative verification of PMVMs’ inherent features of low-torque ripple (<0.2%) and high-average torque (40% larger) when compared to regular PM machines.

The Maxwell stress tensor method is used to show that the improvement of tangential flux density plays a significant role in the high-torque-density feature of VMs [29].

Liu and Zhu [30] provide a more comprehensive air gap flux density analysis with consideration of both FMP and slotting effects. The analytical results show that VMs, such as regular synchronous machines, are mono-harmonic machines, meaning that only one air gap harmonic contributes to torque production. An finite element analysis (FEA)-based equivalent current sheet model has been used for quantitative verification of the radial and tangential flux density spectrum. In [31], expanded analyses reveal that the magnetic gearing effect could be a significant resource of torque production in open-slot FSCW machines.

In [32], a unified bond graph model has been developed for hybrid-excited and PMVMs. A generic power flow-based electric machine model has been highlighted for its fitness for an unconventional machine such as PMVM.

Elaborate analytical modelling of PMVM has been carried out, which can significantly reduce computational time at an early stage of design. Since VMs can have complicated geometries, reluctance network analysis with improved mesh generation methods is used to achieve high accuracy [33]. As a refinement to the existing approach, nonlinearity and end effects have been considered [34].

### **SECTION 3. Torque production analysis through winding function approaches**

Electromagnetic torque of any PMSM is produced by the interaction between stator MMF generated by armature winding and rotor MMF generated by PMs. From this point of view, either the higher stator or rotor MMF is required for achieving a higher torque capability. In addition, PMVMs typically have large slot opening or split teeth, which generally results in larger equivalent air gap length compared to conventional PMSMs with the semi-close slot opening design. Therefore, the outstanding torque capability of PMVMs cannot be fully explained by the magnetic gearing effect. This section

provides a winding-function-based method to analyse and compare MMF production in PMVMs as well as PMSMs, highlighting the impact of no. of pole pairs on MMFs.

### 3.1 Stator-winding-produced MMF

(i) *Integral-slot distributed winding (ISDW)*: As can be seen in Fig. 3, under the assumptions that stator geometry, current density and no. of series turns per phase are identical, the winding-produced MMF and the resulting air gap flux density are inversely proportional to the no. of pole pairs ( $p_s$ ). Besides, winding inductance is inversely proportional to the square of no. of pole pairs, since the area under each pole pair reduces with more poles as well. Therefore, if the rotor MMF remains unchanged, torque is inversely proportional to  $p_s$ .

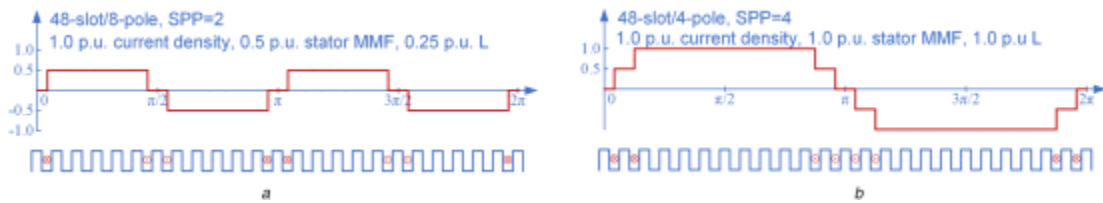
For a low-speed/high-pole-number machine design, VMs generally have lower no. of poles, compare to ISDW PMSM, which allows higher air gap flux density with comparative design assumptions. As shown in Figs. 4 *a* and *b*, both schemes have four split teeth on a major tooth. As the no. of stator-winding pole pairs goes down, winding-produced MMF is getting higher. Moreover, only 0.5 p. u. current density is required for producing that amount of MMF. If the winding is replaced by concentrated tooth coils wound around the split teeth (green parts in Figs. 4), 12 or 20 times amp-turns values are required to achieve the same MMF distribution.

(ii) *FSCW*: As can be seen in Fig. 5, since PMVM can excite four split teeth by one major tooth, the MMF value of FSCW-PMVM is higher than that of FSCW-PMSM.

For the purpose of having the same no. of rotor pole pairs ( $p_r$ ), regular PMSMs have to significantly increase the no. of stator pole pairs ( $p_s$ ), so that the winding-produced MMF and winding inductance will be much lower than their counterparts (PMVMs).

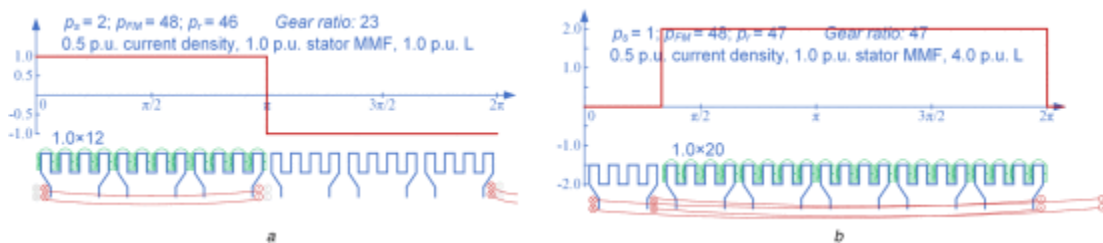
### 3.2 PM-produced MMF

Under the assumption that there are 48 open slots on the stator side, the same air gap length and PM remanence/thickness, Figs. 6 *a–c* show the resulting air gap MMF generated by SPMs (pole arc coeff.: 1.0) in ISDW-PMSM, FSCW-PMSM, and ISDW-PMVM, respectively.



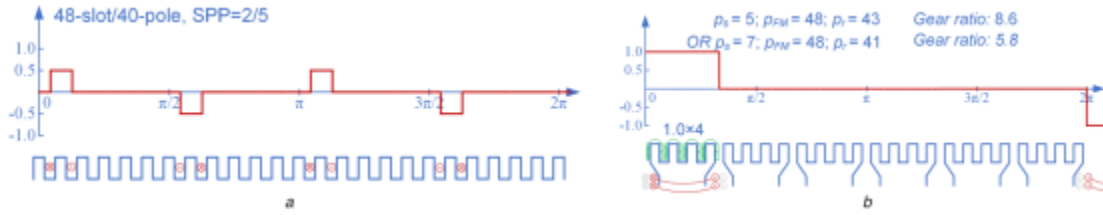
**Fig. 3.** Impact of no. of pole pairs on stator MMF in ISDW-PMSMs

**(a)** 48-slot/eight-pole scheme, **(b)** 48-slot/four-pole scheme (assumptions: both designs have identical current density, no. of series turns and stator geometry, only half of the whole model is shown for simplicity)



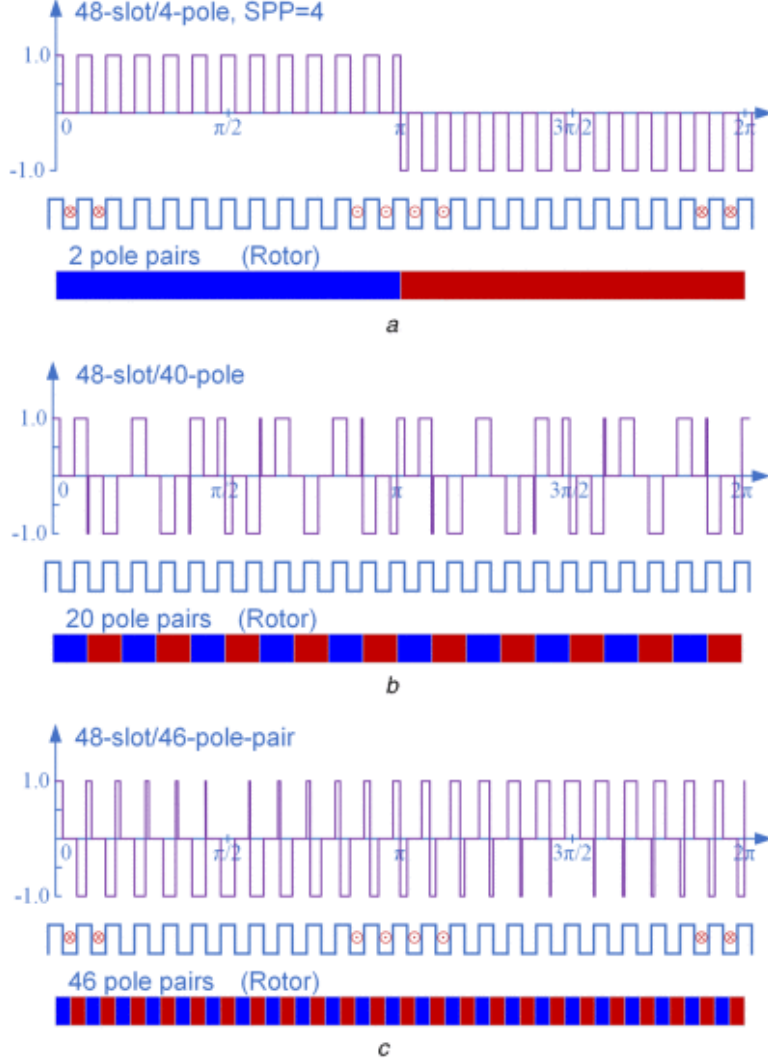
**Fig. 4.** Impact of no. of pole pairs on stator MMF in PMVMs (best viewed in colour online)

(a)  $p_s = 2/p_{FMP} = 48/p_r = 46$ , (b)  $p_s = 1/p_{FMP} = 48/p_r = 47$



**Fig. 5.** Comparison of stator MMFs

(a) 48-slot/40-pole FSCW-PMSM, (b)  $p_s = 5/p_{FMP} = 48/p_r = 43$  or  $p_s = 7/p_{FMP} = 48/$



**Fig. 6.** PM produced MMFs in 48-slot machines

(a) ISDW-PMSM with two pole pairs, (b) FSCW-PMSM with 20 pole pairs, (c) ISDW-PMVM with 46 pole pairs

Unlike winding-produced MMFs, the peak or integral value of PM-produced MMFs are independent of the no. of pole pairs. It is determined by PM remanence, thickness as well as equivalent air gap length. In this regard, all three rotor schemes shown in Figs. 6 a–c can have identical energy (0.5 BH) installed in the magnetic field.



The intensity of the magnetic field generated by rotor sides are similar to each other while the spectra of the PM-generated field are different:

- Little flux modulation effect is shown in either ISDW-PMSM or FSCW-PMSM while a lower-pole-number sinusoidal MMF can be observed in ISDW-PMSM.
- Open-slot design results in low utilisation of rotor PMs. Semi-closed or closed slots are preferred in ISDW-PMSM as well as FSCW-PMSM, which reduces equivalent air gap length, offsetting the disadvantages of regular PMSMs against PMVMs.
- Higher no. of rotor poles generally results in higher flux leakage between two adjacent magnet pieces.

Besides, PMVMs are easier to enter the saturation region than regular PMSMs. For an aggressive design, the advantage of PMVMs can be offset to some extent.

## SECTION 4. Winding, flux modulation pole, and rotor design

This section covers design details of the three components of a VM,

- (i) *Winding patterns*: the two major winding patterns, i.e. ISDWs and FSCWs are adopted in VMs. In addition, DC field windings can be inserted in either the rotor or the stator to substitute for the PMs or achieve hybrid excitation.
- (ii) *FMPs*: the stator teeth of a VM are referred to as the FMPs. Various geometrical designs of FMPs are applied for tuning the MMF spectrum.
- (iii) *Rotor design*: One of the important aspects of rotor design is the choice of PM disposition and shape, where SPM, spoke PM, Halbach PM, consequent pole PM, and hybrid PM are used. Also, there have been designs with salient-pole rotors that place the PMs on the stator.

Table 1 provides a summary of design details regarding the mentioned three components in existing literature.

**Table 1** Summary of design details including winding, FMPs, and rotor

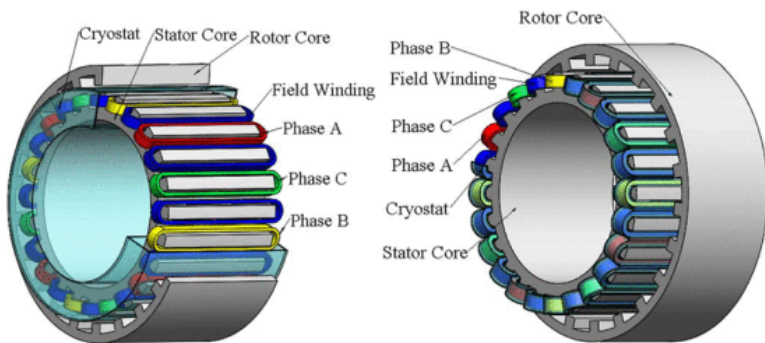
Winding patterns	FMPs	Rotor design
<ul style="list-style-type: none"> <li>• armature winding: ISDW [35], FSCW [36–38];</li> <li>• DC field winding: inserted between teeth [39], in stator slot (toroidal winding) [40], between axially separated stators (parallel-hybrid-excited) [41–43].</li> </ul>	<ul style="list-style-type: none"> <li>• novel FMP shapes: S-shape, concave-shape FMP [44, 45]</li> <li>• non-uniform FMP: adjustable pitch ratio [46], (Fig. 9)</li> <li>• impact of FMP design on radial force: noise &amp; vibration analyses [47, 48]</li> </ul>	<ul style="list-style-type: none"> <li>• SPM: radially-magnetised [49] or Halbach [50, 51]</li> <li>• consequent-pole PM [52]</li> <li>• interior PM: spoke-type [53–59], V-shape, U-shape [60]</li> <li>• hybrid PM (diff. PM materials) [61]</li> </ul>

### 4.1 Winding patterns

The distributed-winding design shows its short-coming with long overlapping end windings [35]. Other alternatives to reduce the copper usage as well as joule losses in the end region have been pursued. One alternative is to use FSCWs [36].

However, the gear ratio and winding factor are in conflict with torque production in an FSCW PMVM. It has also been shown that the flux modulation effect can be offset by a reduction of constant air gap permeance and increased flux leakage. However, fractional-slot PMVMs having two-slot coil pitch are promising candidates in terms of higher gear ratio and winding factor. Their torque capability is lower than that of the distributed-winding PMVMs but still higher than conventional FSCW PM machines [37, 38]. Even though these comparisons are useful, they do not provide insights and a comprehensive evaluation in terms of other major performance metrics such as machine losses and efficiency within a wide speed of range.

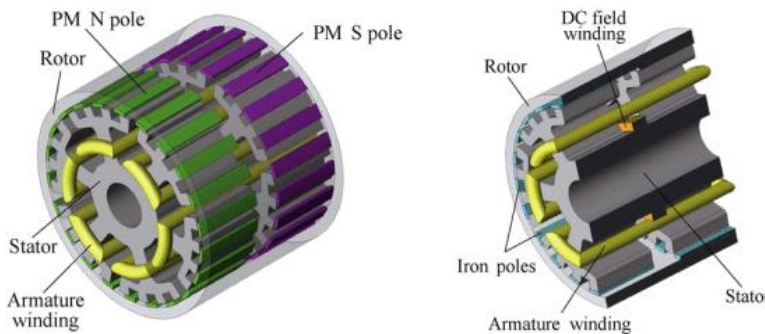
There are also attempts to replace rotor PMs with stator DC windings while the salient-pole rotor poles can be regarded as modulation poles. An example is shown in Fig. 7 where the superconducting DC field windings are wound around the previously non-wound tooth in an alternate-tooth-wound FSCW machine while the outer rotor has salient poles [39].



**Fig. 7.** Cryostat topology for wind power [39]

The toroidal winding has been adopted in a dual-rotor axial-flux PMVM [40]. Compared to concentrated windings and distributed windings, the length of end windings can be significantly reduced.

A parallel-hybrid-excited VM has been proposed [41, 42]. As shown in Fig. 8, the stator is axially separated into two parts to be able to insert a DC coil in between the two parts. A similar structure has been adopted in a linear PMVM to improve power factor [43].



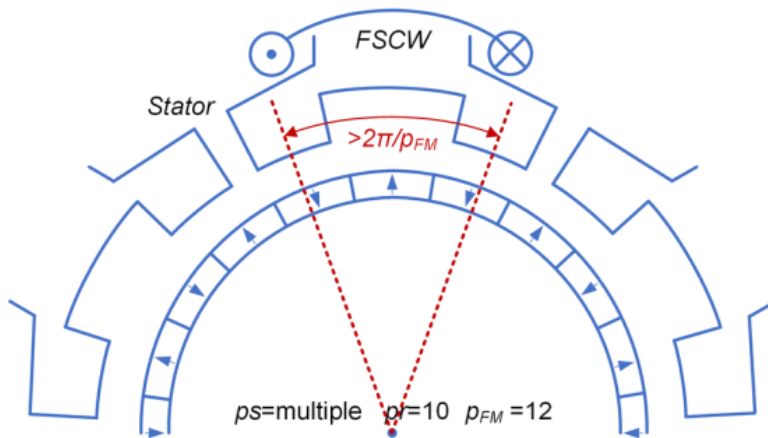
**Fig. 8.** Hybrid-excited homopolar/consequent-pole VM [41]

## 4.2 Flux modulation Poles (FMPs)

The magnetic gearing effect offered by FMPs is a major difference between VMs and conventional synchronous machines. The distribution of the air gap MMF spectrum is determined by the design of FMPs. As a result, leakage inductance and working MMF harmonics can be adjusted based on the design of FMPs.

It appears that PMVMs with concentrated windings suffers more from low-power factor (high-leakage inductance). An S-shaped side line of FMP is proposed based on a multi-objective genetic-algorithm-based optimisation [44]. Torque density and power factor are the two optimisation objectives. It has been shown that the rectangular shape is not the optimal shape for the FMP in a PMVM with the concentrated winding. In addition, a general analytical model has been developed to calculate the power factor of concentrated-winding PMVM with different slot/pole combinations [45].

By adjusting the circumferential position of FMPs, the MMF harmonic spectrum can be regulated. Multi-working harmonics in a six-stator-slot/ten-rotor-pole VM with 12 non-uniform FMPs has been achieved. By adjusting the pitch ratio of FMPs, a torque improvement of 20% has been reported in [46]. Fig. 9 provides a sketch that illustrates the non-uniform distribution of FMPs.



**Fig. 9.** Non-uniform FMP design

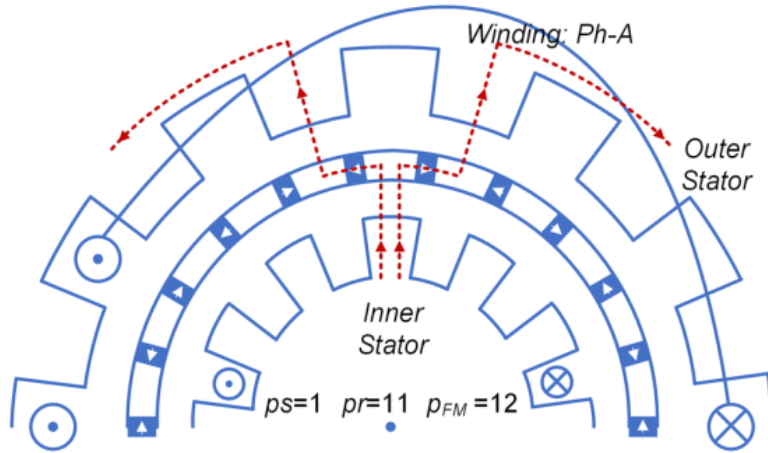
The influence of the FMP width on the radial magnetic force spectrum has been analysed in a six-stator-slot/22-rotor-pole VM with 24 FMPs [47]. The vibration analysis has also been carried out for both distributed-winding and FSCW PMVMs [48]. The proposed methods help in reducing acoustic noise and vibrations during operation.

## 4.3 Rotor design

Many types of PM rotor topologies have been adopted and compared in PMVMs. At the very beginning, SPM rotor was adopted. Although an SPMVM is able to produce 2–3 times the back-electromotive force (EMF) of a regular synchronous PM machine with the same volume, the inductance tends to be much larger, resulting in poor power factor [49]. A typical power factor value for SPMVM could be 0.66 or even lower.

The large winding inductance is primarily due to harmonic leakage components resulting from the flux modulation effect. A double-stator spoke-type PMVM has been proposed to boost power factor, as

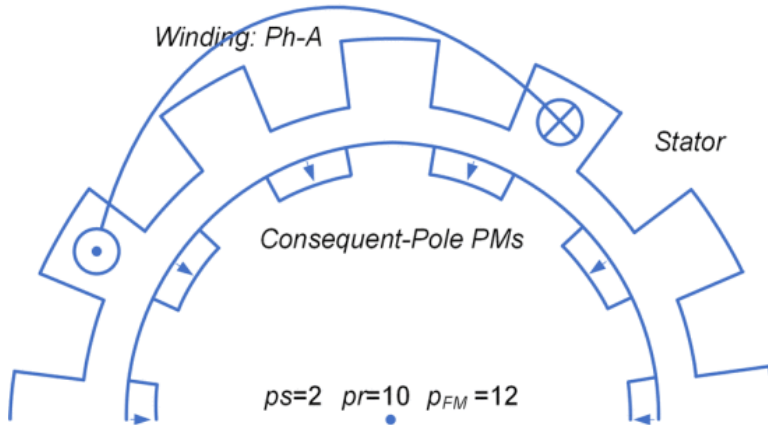
shown in Fig. 10 (schematic diagram). The key enabling design is to have spoke-PM rotors as well as unaligned stators shifted by half tooth pitch that helps in guiding the flux path and achieving a high utilisation of magnets [53–56]. As a complement to this work, Kim and Lipo [57] provide a comprehensive analytical modelling of both single and double air gap spoke-type PMVMs, suggesting that the increased vernier effect can be significantly reduced by the magnetic potential oscillation in the rotor core piece while this issue can be addressed by a design with a double air gap.



**Fig. 10.** Spoke-PM with the double stator

Spoke-type PM with a flux focusing effect can increase the air-gap magnetic loading to improve the power capability of a PMVM [58]. A spoke ferrite magnet VM has been developed for achieving lower active material cost as well as higher power density than its counterpart – a 3 kW, 350 r/min, NdFeB wind turbine synchronous generator [59]. However, it has also been found that the rotor flux barrier of a spoke-type PMVM may weaken the modulated magnetic field as well as torque capability, especially for designs with a high pole ratio ( $p_r/p_s$ ) numbers [52].

Consequent-pole PMVMs have been proposed for reducing rare-earth materials (Fig. 11). An optimised consequent-pole toroidal-winding axial-flux PMVM is able to reach a PM usage reduction of ~66% as well as back EMF/torque boost of ~20%, compared to a regular SPMVM [50]. This is due to the reduced leakage flux as well as interactions with the salient rotor structure.



**Fig. 11.** Consequent-pole design

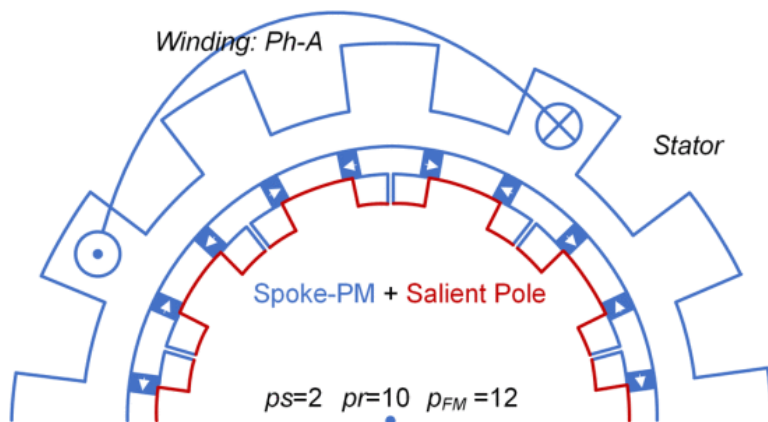
Halbach array helps improve the torque capability of PMVM [51]. One major benefit is the reduced rotor yoke thickness. It has been shown that a topology without yoke iron can reduce the leakage flux albeit with a sacrifice of torque density [61].

The hybrid PM concept, which uses different PM materials in an individual design, helps in reducing cost while meeting performance requirements [62]. Moreover, the rotor MMF harmonics can be reduced without complicated geometrical shaping.

## SECTION 5. Evolution of topologies

To achieve performance improvements specially in terms of torque density, power factor, or cost reduction, complicated magnetic circuit designs have been proposed, resulting in a number of PMVM topologies with compound structures.

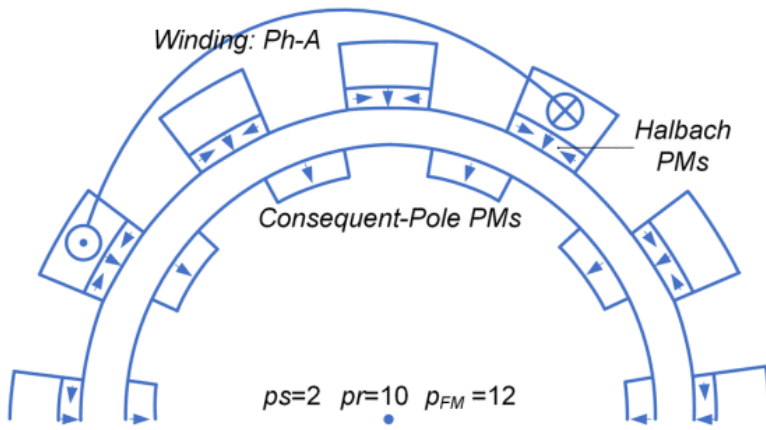
An alternating flux barrier spoke-type VM using ferrite PMs has been proposed [60, 63]. As shown in Fig. 12, the proposed design having an alternating flux barrier below the rotor PMs results in three major rotor magnet flux paths linking with phase A coil. A torque density improvement of 57% has been reported which can even surpass its counterpart with rare-earth PMs. Inherently, the rotor can be segregated as a spoke PM rotor and a salient-pole rotor, each with ten pole pairs.



**Fig. 12.** Alternating flux barrier scheme

Axial flux pattern, spoke-array, and toroidal coil are adopted in a triple-rotor PMVM, enabling dramatic improvements in power factor (0.96) as well as torque density ( $24.2 \text{ kNm/m}^3$ ) [64]. This is primarily attributable to an aggressive usage of PM materials.

As a result of FMPs, PMVMs generally introduce a large slot opening area. Inserting the Halbach PM array at the stator slot opening helps in enhancing the torque capability of the VM (see Fig. 13).

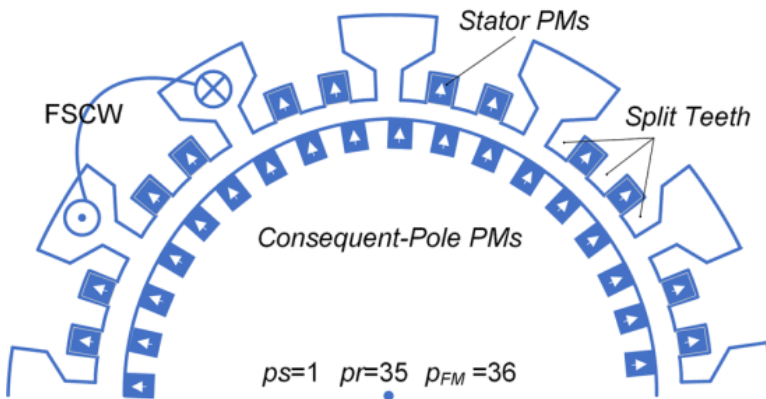


**Fig. 13.** Stator-PM inserted in slot-opening

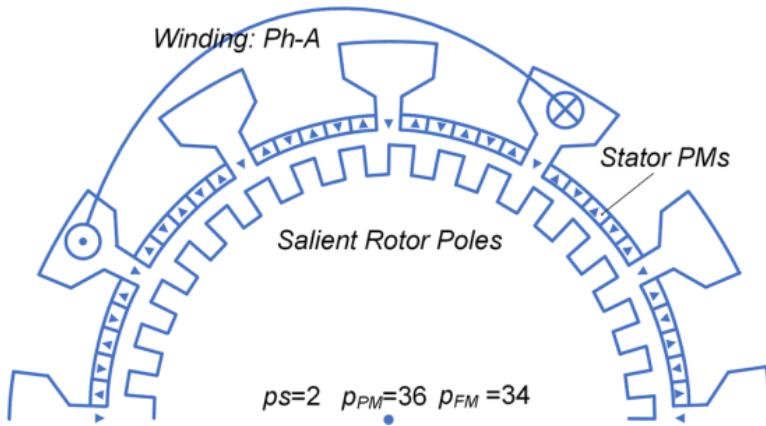
This has been reported in both linear and rotary PMVMs [65–67], in which the Halbach PM array helps in intensifying the main flux path as well as reducing slot leakage flux while the consequent-pole rotor introduces a doubly-salient structure. Therefore, either the stator teeth or the rotor poles can be modulation poles. The back EMF and output torque are dramatically increased compared to a regular PMVM with the same geometry but SPM rotor.

Another example is to add DC coils between the split stator teeth [68]. As a result, adjustable hybrid field excitation is achieved without sacrificing torque density. However, it turns out that this topology suffers from severe temperature rise due to the DC coils [69].

The DC coils between split stator teeth can also be replaced by radially-magnetised PM or Halbach PM array with variable PM arrangements [70–74]. Fig. 14 shows one of those arrangements with the radially-magnetised PM inserted in split teeth. In [75], a comparison of three different armature-side-PM arrangements in a linear VM has been conducted, showing that the circumferential-magnetised arrangements result in a torque improvement of 68% compared to the basic model. The PMs can also be mounted on the surface of the stator tooth (see Fig. 15) [76]. In this case, the salient rotor poles are regarded as the FMPs. It should be noted that Fig. 15 exactly represents a flux reversal machine. The relationship between flux reversal machines and VMs is mentioned in [77].

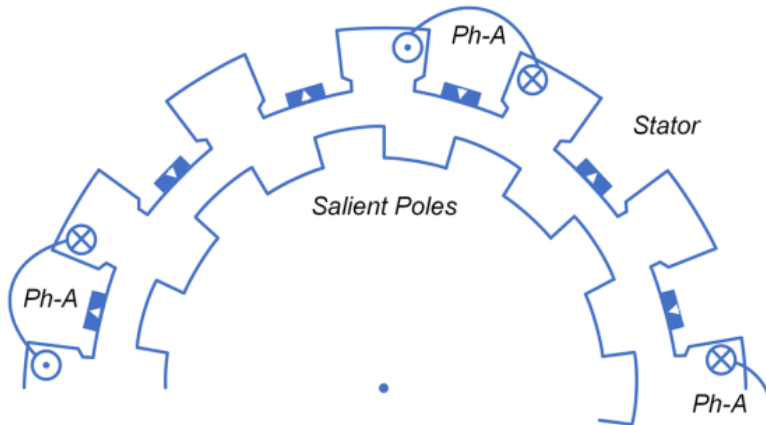


**Fig. 14.** Stator-PM inserted in split-tooth



**Fig. 15.** PM mounted on the tooth surface

A stator-PM, consequent-pole VM with DC-biased sinusoidal current has been reported [78, 79]. Fig. 16 shows that the PMs in this type of machine are buried in the stator tooth pole while the rotor is formed of robust salient ferromagnetic poles. Again, either the stator teeth or the rotor poles can act as modulation poles. In addition, dc-biased current injection helps in reducing PM usage as well as increasing torque density for the same copper losses.



**Fig. 16.** Stator-PM inserted in the tooth

Either radial-flux or axial-flux dual-stator spoke-array PMVMs have been proved to be a promising candidate for torque-density improvements [55, 56, 80]. A key challenge is that heat generated by the inner stator windings can hardly dissipate radially through two air gaps. The inner stator windings can be removed while salient poles are retained in the inner stator. The so-called 'single-winding, dual-stator, spoke-array VM' shows slight reduction in torque density as well as power factor [81, 82]. A similar structure has been adopted in a linear PMVM with reshaped sinusoidal PMs [83].

Some specific applications require high-torque as well as adjustable-speed ability. One way to accomplish this goal is to introduce the flux-variable concept, in which Alnico magnets are adopted for multiple degree-of-freedom magnetisation [84].

Table 2 shows a comparison of different topologies in terms of their torque density, maximum efficiency and power factor. Testing results based on machine prototypes are preferred. Since the

design constraints vary from one design to another, the values included in Table 2 provide a summary of the state-of-the-art PMVMs built and tested in labs rather than an absolute comparison among designs. It can be seen from Table 2 that while very high torque density can be achieved compared to conventional PMSMs, the maximum efficiency and power factor are much lower. This is usually a classical trade-off in most machine topologies but might be more significant in PMVMs.



**Table 2** Summary and comparison of different machine topologies

Topologies	Regular 12-S/22-P [27]	Dual rotor, toroidal coil, axial flux [40]	Non-uniform FMP (Fig. 9) [46]	Double-stator spoke-PM, (Fig. 10) [53]	Consequent pole, toroidal coil (Fig. 11) [50]	Alternating flux barrier, (Fig. 12) [63] <sup>a</sup>	Magnets in stator slot opening [66] (Fig. 13)	Triple-rotor axial-flux spoke-PM [64]	DC-biased current (Fig. 16) [78]
torque density, kNm/m <sup>3</sup>	57 <sup>b</sup>	31.9	20.5	66 <sup>b</sup>	28.6	19.4	23.1	24.2	12.0
maximum efficiency, %	85	90	76.7	85	85	—	—	85	63
power factor	0.6	0.57	0.6	0.83	0.76	0.62	0.89	0.93	0.53

## SECTION 6. Design issues

### 6.1 Gear ratio

In a VM, the gear ratio is defined as the ratio of the number of rotor pole pairs to that of the stator winding pole pairs, representing the magnetic gearing effect. Gear ratio can be an important design index for the choice of slot/pole combinations during the early stages of the machine design [85].

Increasing gear ratio basically enhances torque capability. While high-torque can be achieved with a high number of rotor poles, this typically leads to higher leakage flux. So there is a fundamental trade-off between increasing gear ratio/torque and power factor/leakage flux. Torque ripple of a PMVM is insensitive to design parameters. During design optimisation, it has also been found that the split ratio as well as slot opening ratio increase as gear ratio or winding pole number increases [86].

Design optimisation for PMVMs is supposed to begin with gear ratio analysis while a comprehensive optimisation should take into account stator winding pole number.

### 6.2 Design and optimisation

Due to the multipole configurations, VMs can suffer from severe tooth-tip flux leakage. In [87], an analytical modelling of tooth-tip flux leakage has been developed and verified with numerical results. Then, a practical design procedure for leakage flux reduction can be realised.

Modular design is desired for easy fabrication, replacement, and integration with drive modules. In [88], a VM with modular armature cores has been proposed (Fig. 17). By having two adjacent coils per phase, the stator has been segmented into six modules, which enables fast manufacturing and replacement.



**Fig. 17.** *Assembly of a modular PMVM [88]*

Unlike conventional PM synchronous machines, PMVMs show no or minimum reluctance torque even if they have a salient rotor structure. This is because the rotor saliency is filtered by the flux-modulation effect. Furthermore, this type of machine shows a poor power factor which requires high inverter capacity. Based on this, there have been concerns regarding the CPSR capability of all types of magnetically geared machines. Thus, a hybrid field excitation from both rotor PM and stator DC coils is commonly used for better flux-weakening capability [89].

In [90], the impact of the gear ratio and tooth width ratio on the back EMF has been investigated. The optimisation results show that VMs with a large gear ratio exhibit a remarkable feature of sinusoidal

back EMF. Li *et al.* [91] present the optimisation of stator tooth pole (both depth and width) for pursuing higher thrust force in a linear tubular VM. By adjusting the tooth pole shape, the maximum amplitude of a specific MMF harmonic component can be realised. In [92], Latin hypercube sampling, Kriging method as well as the generic algorithm has been used for the optimal design of a rotor-overhang axial PMVM. Thus, a refined design can be obtained, aiming at a real application.

### 6.3 Feasibility for practical applications

Although PMVMs have many advantages over conventional PMSMs, few practical applications can be seen in the industry at the moment. The potential technical reasons are as follows,

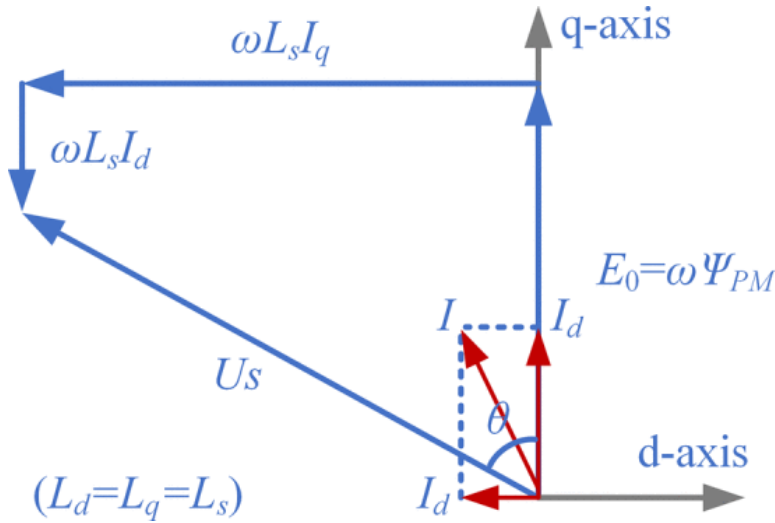
- (i) *Loss of flux modulation effect*: Since direct-drive machines generally have a large size, the air gap length basically gets larger with the increase of machine size. As a result, the flux modulation effect, which is essential for the torque-producing capability of VMs, can be significantly reduced. This prevents PMVMs from being adopted in applications such as large direct-drive wind turbines and turbines for wave energy extraction.
- (ii) *Complicated physical design for power factor improvements*: There are certain ways to address the low-power factor issues of the VMs. However, most of those approaches result in either complicated mechanical structure (e.g. dual stator design) or material processing (e.g. super conducting materials), which is not desired from a manufacturing point of view.
- (iii) *Other design issues*: (i) Most of the PMVMs use SPMs which is not good for protection against demagnetisation; (ii) reluctance torque, which is commonly utilised for achieving wide CPSR operation, cannot be taken advantage of in PMVMs.

Hopefully, those potential challenges can be addressed, and we can see more adoption of PMVMs in commercial applications.

## SECTION 7. Power factor

### 7.1 Reasons for low-power factor

The low-power factor is one of the well-known issues for PMVMs. This issue was pointed out in one of the pioneer papers, in which a typical power factor for a VM can be as low as 0.4 [18]. The understanding of the power factor is usually based on the phasor diagram shown in Fig. 18.



**Fig. 18.** Phasor diagram of a PMVM (stator resistance is neglected for simplicity)

Here the power factor refers to the angle between  $E_0$  and  $U_s$  instead of the angle between  $I$  and  $U_s$ . For a constant torque speed range, these two angles are exactly the same when  $I_d = 0$ . For CPSR when field weakening is applied, the angle between  $I$  and  $U_s$  may be lower (power factor is higher). When  $I_d = 0$ , the definition of the power factor is given as follows:

$$\cos \theta_{\text{ver}} = \frac{1}{\sqrt{1 + (L_s I / \Psi_{PM})^2}}, \quad (3)$$

where  $L_s$  is the synchronous inductance,  $I$  is the phase current, and  $\Psi_{PM}$  is the PM-generated flux linkage.

The low-power factor feature of a PMVM can be mainly explained by

(i) *Poor utilisation of PMs and leakage flux:* As can be seen from Fig. 6 c, for a typical PMVM, only half of the PMs contributes to the synchronous rotor MMF while the other half produce leakage components, which jointly results in high-winding inductance ( $L_s$ ) and low PM flux linkage ( $\Psi_{PM}$ ) [53].

The stator teeth, which act as FMPs in a PMVM, feature wide slot opening, resulting in low Carter's coefficient (large equivalent air gap length). In addition, rotor pole numbers are usually high. Therefore, the flux leakage between two adjacent magnets is relatively high.

(ii) *High-electrical frequency and high slot per pole per phase number:* Compared to a conventional PMSM, the increased number of rotor PMs in a PMVM leads to high-electrical frequency, and therefore a much higher inductive reactance [49]. The relationship of the power factor angle for PMVMs ( $\theta_{\text{ver}}$ ) and PMSMs ( $\theta_c$ ) is given as follows:

$$\cos \theta_{\text{ver}} = \frac{1}{\sqrt{1 + (p_r / p_s k_e)^2 \tan^2 \theta_c}} = \frac{1}{\sqrt{1 + (6q - 1 / k_e)^2 \tan^2 \theta_c}}, \quad (4)$$

where  $p_r = p_s (6q - 1)$  ( $q$  is the slot per pole per phase number);  $k_e$  is the coefficient of voltage boost which represents the voltage boost capability of PMVMs over conventional PMSMs.

Based on (4), it is clear that the power factor is significantly reduced with a higher value of  $q$ . It is recommended to have  $q < 3$  in order to achieve a power factor  $> 0.7$ . Obviously, this will limit the maximum gear ratio that can be achieved.

## 7.2 Approaches to improve power factor

Promising design approaches to improve the power factor of PMVMs are summarised as follows.

- (i) *Halbach PM array*: By eliminating ferromagnetic rotor back iron while keeping pretty much the same PM flux, the magnetisation inductance can be significantly reduced. In [93], a power factor value of 0.73 is supported by FEA albeit with a sacrifice of torque by  $\sim 50\%$ .
- (ii) *Dual stator (air gaps) with surface/spoke PMs*: SPM machines with the double stator, which can be regarded as a combination of a regular PMVM and an inside-out PMVM, can achieve a higher power factor. This is due to lower electric loading as well as low slot rate permeance. In [54], a power factor value of 0.85 is reported by FEA.
- Having inner and outer stators offset by a half slot pitch relative to the spoke PM rotor that helps in guiding the flux path and achieving a high utilisation of magnets [26]. Testing results show that a power factor of 0.83 can be achieved [53].
- (iii) *Additional dc field winding*: An additional dc field winding has been used in a paralleled-hybrid-excited linear PMVM for wave power generation, which can compensate for the PM-generated flux [42, 89]. As a result, back-EMF ( $E_o$ ) can be boosted without an increase of PM thickness. Claimed by the paper, the power factor for generating and motoring modes can be improved from  $\sim 0.5$  to  $\sim 0.8$ . However, this design results in a sacrifice of machine efficiency ( $\sim 12\%$ ) due to copper losses in the dc field winding.
- (iv) *Selection of slot/pole combinations and winding arrangements*: By purposely choosing slot/pole combinations as well as winding arrangements, winding MMF harmonics can be reduced so that a lower inductance is achieved without affecting the fundamental component. FSCW with two coil pitches have been proposed for improving power factor [94]. Measurements on the 18-slot/14-pole and 18-slot/26-pole PMVM show that the power factor can be improved to 0.91 and 0.92, respectively [94, 95]. The design principle is given in [96]. Again, it should be noted that a sacrifice of torque capability is expected compared to a typical PMVM design.

As a brief conclusion, all those approaches will result in either a complicated mechanical structure (A#2) or degradation of machine performance, i.e. torque capability (A#1, A#4) or efficiency (A#3). Finally, offset the advantages of PMVMs to some extent.

## SECTION 8 Advanced materials

The importance of advanced materials has been more recognised especially in the design of VMs as well as other members of the MGM family. An overview of the potential merits of advanced materials and manufacturing in machine design is given by El-Refaie [97]. It should be noted that cost and availability could always be an issue for advanced materials.

### 8.1 High-temperature superconductor (HTS)

To reduce the leakage flux in stator tooth, a design with high-temperature superconductor (yttrium barium copper oxide) bulks inserted between modulation poles is reported [98–101]. The inherent

magnetic shielding effect of HTS materials results in a significant power factor improvement from 0.4 to 0.6 in a double-stator spoke-type linear PMVM for wave energy extraction. Besides, thrust force density is increased by 34% albeit with higher force ripple as well as joule losses. A similar investigation has been conducted in a tubular PM linear VM.

The HTS material has also been used to build rotor field winding as well as stator windings in VMs for megawatt direct-drive wind turbines [102]. Advantages include high efficiency, high-torque density, and low-short-circuit current. Another example is to use the HTS DC winding in parallel-hybrid-excited VMs. Since the DC winding is placed between two segmented stators, this helps to reduced copper losses as well as heat generated by the DC winding [103].

## 8.2 Dual-phase magnetic material

Generally speaking, magnetic gear and magnetically geared machines suffer from higher flux leakage as well as a low-power factor. This is due to their compound structures with either flux modulator poles or a variety of split teeth designs. Since a PMVM is specifically designed for high-torque-density applications, interior PMs with flux concentration effects, such as spoke-type, V-shape and U-shape PMs are adopted. Like many of the conventional IPM synchronous machines, PMVM design is subject to a trade-off between mechanical strength and leakage flux through rotor bridges.

Since the dual-phase magnetic materials can selectively introduce magnetic or non-magnetic regions within the lamination, lowering leakage flux while maintaining good mechanical strength becomes possible [104, 105], this can be applied in the design of modulation poles as well as spoke-type PMs which leads to significantly-reduced leakage flux as well as compact design with good mechanical strength.

## SECTION 9 Reliability and fault tolerance

Reliability is of great significance for efforts push to adopt VMs in commercial applications. The fault-tolerance-related knowledge learned from conventional PM machines and drives can be transferred to the analysis and design of PMVMs.

- (i) *Multi-phase PM VMs*: Like many other electrical machines, multi-phase windings have been introduced in PMVMs for safety-critical applications [106, 107]. Thus, phase redundancy offers opportunities for post-fault operation when failures occur in one or even more phases. In addition, alternate-tooth-wound FSCWs are adopted to improve the isolation features [108, 109]. Among all types of winding failures, winding short circuits (terminal short circuit or inter-turn short circuit) is the most severe scenarios. A current phase-angle control method has been developed as a post-fault control strategy for a 20-stator-slot/62-rotor-pole/40-FMP outer rotor PMVM with winding terminal short-circuit fault [110].
- (ii) *Sensorless/self-sensing*: Eliminating mechanical rotor position sensors such as revolvers, encoders and Hall-effect sensors has become a general consideration to improve the overall system reliability. Either back-EMF based or high-frequency-injection based sensorless/self-sensing control has been carried out for PMVM drives. Since PMVMs generally have a low-winding pole number, they can easily get saturated even with low-winding current density. Therefore, nonlinearity caused by saturation and cross coupling have to be taken into account to improve position estimation accuracy [111–113].

- (iii) *Demagnetisation*: As one of the issues that might occur under fault and/or PM overheating, demagnetisation has also been receiving lot of attention. This issue is especially important for low-cost designs that use low-coercive-force magnets. To provide insights into the relationship between design parameters and demagnetisation risks, analytical modelling has been carried out to achieve the maximum electrical and magnetic loading while taking demagnetisation risk into consideration [114].

## SECTION 10 Conclusion

This overview reflects the growing interest and understanding of VMs in recent years despite the fact that they have been invented several decades ago. It has been revealed in this study that one important aspect of PMVM's outstanding torque capability is based on the choice of low no. of pole pairs on the stator side. Sustained efforts to develop novel optimised designs enabled by advanced materials will enable reach performance, reliability and cost targets and hence open the door for large-scale adoption of VMs in a wide range of applications. The hope is that this study will serve as a good reference for researchers working in this field that provides them with a comprehensive overview of the state-of-the-art, research efforts as well as technology gaps.

## References

1. D. Li, R. Qu and J. Li, "Topologies and analysis of flux-modulation machines", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 2153-2160, September 2015.
2. W.N. Fu and Y. Liu, "A unified theory of flux-modulated electric machines", *Proc. Int. Symp. on Electrical Engineering*, pp. 1-13, December 2016.
3. F. Wu and A.M. El-Refaie, "Permanent magnet vernier machines: a review", *IEEE ICEM*, pp. 372-378, 2018.
4. P. Zheng, J. Bai, C. Tong et al., "Investigation of a novel radial magnetic-field-modulated brushless double-rotor machine used for HEVs", *IEEE Trans. Magn.*, vol. 49, no. 3, pp. 1231-1241, 2013.
5. C.H. Lee, "Vernier motor and its design", *IEEE Trans. Power Appl. Syst.*, vol. 82, no. 66, pp. 343-349, 1960.
6. A. Ishizaki, T. Tanaka, K. Takahashi et al., "Theory and optimum design of PM vernier motor", *Proc. IEEE Int. Conf. on Electrical Machines and Drives*, pp. 208-212, September 1995.
7. A. Toba and T.A. Lipo, "Generic torque-maximizing design methodology of surface permanent-magnet vernier machine", *IEEE Trans. Ind. Appl.*, vol. 36, no. 6, pp. 1539-1546, 2000.
8. A. Toba and T. A. Lipo, "Novel dual-excitation permanent magnet vernier machine", *Conf. Records of the IEEE IAS Annual Meeting*, pp. 2539-2544, October 1999.
9. P.R.M. Brooking and M. A. Mueller, "Power conditioning of the output from a linear vernier hybrid permanent magnet generator for use in direct drive wave energy converters", *IEE Proc. Gener. Transm. Distrib.*, vol. 152, no. 5, pp. 673-681, 2005.
10. J. Li, K.T. Chau, J.Z. Jiang et al., "A new efficient permanent-magnet vernier machine for wind power generation", *IEEE Trans. Magn.*, vol. 46, no. 6, pp. 1475-1478, 2010.
11. M.-S. Kim, F. Zhao and B.-I. Kwon, "Design and analysis of an axial-flux PM vernier machine for auto-focusing systems", *Proc. IEEE ICEMS*, pp. 1792-1796, October 2015.
12. F. Zhao, M.-S. Kim, B.-I. Kwon et al., "A small axial-flux vernier machine with ring-type magnets for the auto-focusing lens drive system", *IEEE Trans. Magn.*, vol. 52, no. 7, 2016.

13. C. Bode, H. Schillingmann and M. Henke, "A free-piston PM linear generator in vernier topology using quasi-Halbach-excitation", *Proc. IEEE Int. Conf. on Electrical Machines*, pp. 1950-1955, September 2014.
14. Y. Oner, Z.Q. Zhu and W. Chu, "Comparative study of vernier and interior PM machines for automotive application", *Proc. IEEE Vehicle Power and Propulsion Conf.*, pp. 1-6, October. 2016.
15. S. Gerber and R. Wang, "Design and evaluation of a PM vernier machine", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 5188-5194, September 2015.
16. C. Espanet, J.-M. Kauffmann and R. Bernard, "Comparison of two in-wheel permanent magnet motors for military applications", *Proc. IEEE Vehicle Power and Propulsion Conf.*, pp. 1-6, September 2006.
17. J. Li and K.T. Chau, "Performance and cost comparison of permanent magnet vernier machines", *IEEE Trans. Appl. Supercond.*, vol. 22, no. 3, 2012.
18. E. Spooner and L. Hardock, "Vernier hybrid machines", *IEE Proc. Electr. Power Appl.*, vol. 150, no. 6, pp. 655-662, 2003.
19. B. Kim, "Characteristic analysis of a vernier PM motor considering adjustable speed control", *Proc. IEEE Transportation Electrification Conf.*, pp. 671-676, June 2016.
20. S. Tabi, A. Tounzi and F. Piriou, "Study of a stator current excited vernier reluctance machine", *IEEE Trans. Energy Convers.*, vol. 21, no. 4, pp. 823-831, 2006.
21. S. Jia, R. Qu, J. Li et al., "Principles of stator DC winding excited vernier reluctance machines", *IEEE Trans. Energy Convers.*, vol. 31, no. 3, pp. 935-946, 2016.
22. L. Xu, G. Liu and W. Zhao, "Stator-excited vernier high-temperature superconducting machine for direct drive propulsion", *IEEE Trans. Appl. Supercond.*, vol. 26, no. 7, 2016.
23. S. Jia, R. Qu, J. Li et al., "Comparison of stator DC-excited vernier reluctance machines with synchronous reluctance machines", *Proc. IEEE Int. Electric Machines & Drives Conf.*, pp. 649-655, May 2015.
24. D. Fukai and S. Shimomura, "Integrated radial and dual axial-flux variable-reluctance vernier machine", *Proc. IEEE IECON*, pp. 682-688, October/November 2014.
25. D. Li, R. Qu and J. Wang, "Relationship between magnetic gears and vernier machines", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-6, August 2011.
26. T.A. Lipo and B. Kim, "Double air gap spoke type vernier machine", Mar. 14, 2017.
27. D. Li, R. Qu, J. Li et al., "Analysis of torque capability and quality in vernier permanent-magnet machines", *IEEE Trans. Ind. Appl.*, vol. 52, no. 1, pp. 125-135, 2016.
28. L. Wu, R. Qu, D. Li et al., "Power transferring of magnetic-gear permanent magnet machines", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 1-7, September 2016.
29. K. Xie, D. Li, R. Qu et al., "A new perspective on the PM vernier machine mechanism", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 5001-5008, October 2017.
30. Y. Liu and Z.Q. Zhu, "Magnetic gearing effect in vernier permanent magnet synchronous machines", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 5025-5032, October 2017.
31. Z.Q. Zhu and Y. Liu, "Analysis of air-gap field modulation and magnetic gearing effect in fractional slot concentrated winding permanent magnet synchronous machines", *IEEE Trans. Ind. Electron.*, vol. 65, no. 5, pp. 3688-3698, 2018.



32. G. Komurgoz and G.-H. Geitner, "Application of the unified electric motor bond graph model to HB-type and PM-type vernier machines", *Proc. IEEE IECON*, pp. 4263-4268, October 2016.
33. G. Liu, S. Jiang, W. Zhao et al., "Modular reluctance network simulation of a linear permanent-magnet vernier machine using new mesh generation methods", *IEEE Trans. Ind. Electron.*, vol. 64, no. 7, pp. 5323-5332, 2017.
34. G. Liu, L. Ding, W. Zhao et al., "Nonlinear equivalent magnetic network of a linear permanent magnet vernier machine with end effect consideration", *IEEE Trans. Magn.*, vol. 54, no. 1, 2018.
35. B. Kim and T.A. Lipo, "Design of a surface PM vernier motor for a practical variable speed application", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 776-783, September 2015.
36. J. Li, D. Wu, X. Zhang et al., "A new permanent-magnet vernier in-wheel motor for electric vehicles", *Proc. IEEE Vehicle Power and Propulsion Conf.*, pp. 1-6, September 2010.
37. T. Zou, R. Qu, D. Li et al., "Synthesis of fractional-slot vernier permanent magnet machines", *Proc. IEEE Int. Conf. on Electrical Machines*, pp. 911-917, September 2016.
38. L. Xu, G. Liu, W. Zhao et al., "Quantitative comparison of integral and fractional slot permanent magnet vernier motors", *IEEE Trans. Energy Convers.*, vol. 30, no. 4, pp. 1483-1495, 2015.
39. S. Jia, R. Qu, J. Li et al., "A novel vernier reluctance fully superconducting direct drive synchronous generator with concentrated windings for wind power application", *IEEE Trans. Appl. Supercond.*, vol. 26, no. 7, 2016.
40. T. Zou, D. Li, R. Qu et al., "Analysis of a dual-rotor toroidal-winding axial-flux vernier permanent magnet machine", *IEEE Trans. Ind. Appl.*, vol. 53, no. 3, pp. 1920-1930, 2017.
41. W. Li, T.W. Ching and K. T. Chau, "Design and analysis of an outer-rotor parallel-hybrid-excited vernier machine", *Chin. J. Electr. Eng.*, vol. 3, no. 1, pp. 27-32, 2017.
42. W. Li, T.W. Ching and K.T. Chau, "A hybrid-excited vernier permanent magnet machine using homopolar topology", *IEEE Trans. Magn.*, vol. 53, no. 11, 2017.
43. T.W. Ching, K.T. Chau and W. Li, "Power factor improvement of a linear vernier permanent-magnet machine using auxiliary DC field excitation", *IEEE Trans. Magn.*, vol. 52, no. 7, 2016.
44. D. Thyroff and I. Hahn, "Optimization of the flux modulation Poles for vernier machines with concentrated windings", *Proc. IEEE WEMDCD*, pp. 113-118, April 2017.
45. D. Thyroff, C. Hittinger and I. Hahn, "Analytic power factor calculation for vernier machines with concentrated windings", *Proc. IEEE Int. Electric Machines and Drives Conf.*, pp. 1-6, May 2017.
46. T. Zou, D. Li, R. Qu et al., "Advanced high torque density PM vernier machine with multiple working harmonics", *IEEE Trans. Ind. Appl.*, vol. 53, no. 6, pp. 5295-5304, 2017.
47. D.-K. Jang and J.-H. Chang, "Effects of flux modulation Poles on the radial magnetic forces in surface-mounted permanent-magnet vernier machines", *IEEE Trans. Magn.*, vol. 53, no. 6, 2017.
48. L. Wu, R. Qu, H. Fang et al., "Radial force and vibration in surface-mounted permanent magnet vernier machines", *Proc. IEEE Int. Conf. on Electrical Machines*, pp. 2328-2334, September 2016.
49. B. Kim and T.A. Lipo, "Operation and design principles of a PM vernier motors", *IEEE Trans. Ind. Appl.*, vol. 50, no. 6, pp. 3656-3663, 2014.
50. D. Li, R. Qu, J. Li et al., "Consequent-pole toroidal-winding outer-rotor vernier permanent-magnet machines", *IEEE Trans. Ind. Appl.*, vol. 51, no. 6, pp. 4470-4481, 2015.

51. Y. Kataoka, M. Takayama, Y. Matsushima et al., "Design of surface permanent magnet-type vernier motor using Halbach array magnet", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 177-183, October 2015.
52. T. Zou, D. Li, R. Qu et al., "Performance comparison of surface and spoke-type flux-modulation machines with different pole ratios", *IEEE Trans. Magn.*, vol. 53, no. 6, pp. 1-5, 2017.
53. D. Li, R. Qu and T.A. Lipo, "High-power-factor vernier permanent magnet machines", *IEEE Trans. Ind. Appl.*, vol. 50, no. 6, pp. 3664-3674, 2014.
54. D. Li, R. Qu and Z. Zhu, "Comparison of Halbach and dual-side vernier permanent magnet machines", *IEEE Trans. Magn.*, vol. 50, no. 2, 2014.
55. F. Zhao, T.A. Lipo and B.-I. Kwon, "A novel dual-stator axial-flux spoke-type permanent magnet vernier machine for direct-drive applications", *IEEE Trans. Magn.*, vol. 50, no. 11, 2014.
56. D. Li, R. Qu, W. Xu et al., "Design procedure of dual-stator spoke-array vernier permanent-magnet machines", *IEEE Trans. Ind. Appl.*, vol. 51, no. 4, pp. 2972-2983, 2015.
57. B. Kim and T.A. Lipo, "Analysis of a PM vernier motor with spoke type structure", *IEEE Trans. Ind. Appl.*, vol. 52, no. 1, pp. 217-225, 2016.
58. X. Li, K.T. Chau and M. Cheng, "Comparative analysis and experimental verification of an effective permanent-magnet vernier machine", *IEEE Trans. Magn.*, vol. 51, no. 7, 2015.
59. J.F. Kölzer, T. Bazzo, R. Carlson et al., "Comparative analysis of a spoke ferrite permanent magnets vernier synchronous generator", *Proc. IEEE Int. Conf. on Electrical Machines*, pp. 218-224, September 2016.
60. W. Liu and T.A. Lipo, "A family of vernier permanent magnet machines utilizing an alternating rotor leakage flux blocking design", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 2461-2468, October 2017.
61. L. Xu, G. Liu, W. Zhao et al., "High-performance fault tolerant Halbach permanent magnet vernier machines for safety-critical applications", *IEEE Trans. Magn.*, vol. 52, no. 7, 2016.
62. G. Liu, X. Fan, W. Zhao et al., "Analysis of magnet material effects on performances of fault-tolerant PM vernier machines", *IEEE Trans. Appl. Supercond.*, vol. 26, no. 7, 2016.
63. W. Liu and T.A. Lipo, "Analysis of consequent pole spoke type vernier permanent magnet machine with alternative flux barrier design", *IEEE Trans. Ind. Appl.*, vol. 54, no. 6, pp. 5918-5929, 2018.
64. R. Zhang, J. Li, R. Qu et al., "Analysis and design of triple-rotor axial-flux spoke-array vernier permanent magnet machines", *IEEE Trans. Ind. Appl.*, vol. 54, no. 1, pp. 244-253, 2018.
65. Y. Kataoka, M. Takayama, Y. Matsushima et al., "Design of high torque PM vernier motor", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-6, November 2016.
66. C. Shi, D. Li, R. Qu et al., "A novel linear permanent magnet vernier machine with consequent-pole permanent magnets and Halbach permanent magnet arrays", *IEEE Trans. Magn.*, vol. 53, no. 11, 2017.
67. K. Xie, D. Li, R. Qu et al., "A novel permanent magnet vernier machine with Halbach array magnets in stator slot opening", *IEEE Trans. Magn.*, vol. 53, no. 6, 2017.
68. H. Wang, S. Fang, H. Yang et al., "A novel consequent-pole hybrid excited vernier machine", *IEEE Trans. Magn.*, vol. 53, no. 11, 2017.
69. H. Wang, S. Fang, H. Yang et al., "Loss calculation and temperature field analysis of consequent-pole hybrid excited vernier machine", *IEEE Trans. Magn.*, vol. 53, no. 11, 2017.

70. W. Zhao, X. Sun, J. Ji et al., "Design and analysis of new vernier permanent-magnet machine with improved torque capability", *IEEE Trans. Appl. Supercond.*, vol. 26, no. 4, 2016.
71. Y. Du, M. Cheng, K.T. Chau et al., "Linear primary permanent magnet vernier machine for wave energy conversion", *IET Electr. Power Appl.*, vol. 9, no. 3, pp. 203-212, 2015.
72. D.-K. Jang and J.-H. Chang, "Design of a vernier machine with PM on both sides of rotor and stator", *IEEE Trans. Magn.*, vol. 50, no. 2, 2014.
73. S. Niu, S.L. Ho and W.N. Fu, "A novel stator and rotor dual PM vernier motor with space vector pulse width modulation", *IEEE Trans. Magn.*, vol. 50, no. 2, 2014.
74. A. Kohara, K. Hirata, N. Niguchi et al., "Finite-element analysis and experiment of current superimposition variable flux machine using permanent magnet", *IEEE Trans. Magn.*, vol. 52, no. 9, 2016.
75. T. Imada and S. Shimomura, "Magnet arrangement of linear PM vernier machine", *Proc. IEEE Int. Conf. on Electrical Machines and Systems.*, pp. 3642-3646, October 2014.
76. D. Yi, M. Cheng, K.T. Chau et al., "Comparison of linear primary permanent magnet vernier machine and linear vernier hybrid machine", *IEEE Trans. Magn.*, vol. 50, no. 11, 2014.
77. H.Y. Li, Y. Liu and Z.Q. Zhu, "Comparative study of airgap field modulation in flux reversal and vernier permanent magnet machines", *IEEE Trans. Mag.*, vol. 54, no. 11, pp. 1-6, 2018.
78. S. Jia, R. Qu, W. Kong et al., "Hybrid excited vernier PM machines with novel DC-biased sinusoidal armature current", *IEEE Trans. Ind. Appl.*, vol. 54, no. 2, pp. 1339-1348, 2018.
79. S. Jia, R. Qu, J. Li et al., "A stator-PM consequent-pole vernier machine with hybrid excitation and DC-biased sinusoidal current", *IEEE Trans. Magn.*, vol. 53, no. 6, 2017.
80. D. Li, R. Qu and T.A. Lipo, "High power factor vernier permanent magnet machines", *IEEE Trans. Ind. Appl.*, vol. 50, no. 6, pp. 3664-3674, 2014.
81. D. Li, R. Qu and J. Li, "Development and experimental evaluation of a single-winding dual-stator spoke-array vernier permanent magnet machines", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 1885-1891, September. 2015.
82. M. Raza, W. Zhao, T.A. Lipo et al., "Performance comparison of dual airgap and single airgap spoke-type permanent-magnet vernier machines", *IEEE Trans. Magn.*, vol. 53, no. 6, 2017.
83. W. Li and T.W. Ching, "A new segmented-stator linear vernier permanent magnet machine for direct-drive applications", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-6, August 2017.
84. H. Yang, H. Lin, Z.Q. Zhu et al., "Novel flux-regulatable dual-magnet vernier memory machines for electric vehicle propulsion", *IEEE Trans. Appl. Supercond.*, vol. 24, no. 5, 2014.
85. Y. Liu and Z.Q. Zhu, "Influence of gear ratio on electromagnetic performance and geometries of vernier permanent magnet synchronous machines", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 2453-2460, October 2017.
86. L. Wu, R. Qu, D. Li et al., "Influence of pole ratio and winding pole numbers on performance and optimal design parameters of surface permanent-magnet vernier machines", *IEEE Trans. Ind. Appl.*, vol. 51, no. 5, pp. 3707-3715, 2015.
87. W. Li, K.T. Chau, C. Liu et al., "Analysis of tooth-tip flux leakage in surface-mounted permanent magnet linear vernier machines", *IEEE Trans. Magn.*, vol. 49, no. 7, pp. 3949-3952, 2013.

88. S.-U. Chung, J.-W. Kim, B.-C. Woo et al., "A novel design of modular three-phase permanent magnet vernier machine with consequent pole rotor", *IEEE Trans. Magn.*, vol. 47, no. 10, pp. 4215-4218, 2011.
89. C. Liu, J. Zhong and K.T. Chau, "A novel flux-controllable vernier permanent-magnet machine", *IEEE Trans. Magn.*, vol. 47, no. 10, pp. 4238-4241, 2011.
90. D. Li and R. Qu, "Sinusoidal back-EMF of vernier permanent magnet machines", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-6, October 2012.
91. W. Li, K.T. Chau and C.H.T. Lee, "Optimal design and implementation of a permanent magnet linear vernier machine for direct drive wave energy extraction", *Proc. IEEE IECON*, pp. 6200-6205, October 2012.
92. F. Zhao, L. Li, C. Liu et al., "Design considerations and performance improvement of a dual-stator PM vernier motor with axial-flux loop", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 1220-1226, October. 2017.
93. Y. Kataoka, M. Takayama, Y. Matsushima et al., "Comparison of three magnet array-type rotors in surface permanent magnet-type vernier motor", *Proc. Int. Conf. on Electrical Machines and Systems*, pp. 1-6, October 2012.
94. D. Li, T. Zou, R. Qu et al., "Analysis of fractional-slot concentrated winding PM vernier machines with regular open-slot stators", *IEEE Trans. Ind. Appl.*, vol. 54, no. 2, pp. 1320-1330, 2018.
95. Y. Liu, H.Y. Li and Z.Q. Zhu, "A high-power factor vernier machine with coil pitch of two slot pitches", *IEEE Trans. Magn.*, vol. 54, no. 11, pp. 1-5, 2018.
96. Y. Liu and Z.Q. Zhu, "Electromagnetic performance comparison of 18-slot/26-pole and 18-slot/10-pole fractional slot permanent magnet surface-mounted machines", *Proc. 20th Int. Conf. on Electrical Machines and Systems*, pp. 1-6, August 2017.
97. A.M. El-Refaie, "Role of advanced materials in electrical machines", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-6, August 2017.
98. J. Li and K.T. Chau, "A novel HTS PM vernier motor for direct-drive propulsion", *IEEE Trans. Appl. Supercond.*, vol. 21, no. 3, pp. 1175-1179, 2011.
99. Y. Du, K.T. Chau, M. Cheng et al., "A linear stator permanent magnet vernier HTS machine for wave energy conversion", *IEEE Trans. Appl. Supercond.*, vol. 22, no. 3, 2012.
100. N. Baloch, S. Khaliq and B.-I. Kwon, "HTS dual-stator spoke-type linear vernier machine for leakage flux reduction", *IEEE Trans. Magn.*, vol. 53, no. 11, 2017.
101. N. Baloch, S. Khaliq and B.-I. Kwon, "A high force density HTS tubular vernier machine", *IEEE Trans. Magn.*, vol. 53, no. 11, 2017.
102. G. Yu, R. Qu, J. Li et al., "HTS vernier machine for direct-drive wind power generation", *IEEE Trans. Appl. Supercond.*, vol. 24, no. 5, 2014.
103. W. Li, T.W. Ching and K.T. Chau, "A new high-temperature super-conducting vernier permanent-magnet machine for wind turbines", *IEEE Trans. Appl. Supercond.*, vol. 27, no. 4, 2017.
104. P.B. Reddy, S. Galioto, A.M. El-Refaie et al., "Design of synchronous reluctance motor utilizing dual-phase material for traction applications", *IEEE Trans. Ind. Appl.*, vol. 53, no. 3, pp. 1948-1957, 2017.
105. P.B. Reddy, A.M. El-Refaie, M. Zou et al., "Performance testing and analysis of synchronous reluctance motor utilizing dual-phase magnetic material", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-8, August 2017.

106. M. Wefky, A.S.A. Khalik, S. Ahmed et al., "Performance evaluation of five-phase outer-rotor permanent magnet vernier machines", *Proc. IEEE IECON*, pp. 1852-1857, October 2016.
107. Y. Shen, M. Kang, J. Ji et al., "Design and analysis of a novel modular six-phase linear permanent-magnet vernier machine", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1-5, August 2017.
108. Z. Fang, W. Zhao, J. Ji et al., "A primary-permanent-magnet vernier linear machine with improved fault-tolerant capability", *Proc. IEEE Int. Conf. on Electrical Machines and Systems*, pp. 1882-1885, October 2013.
109. G. Liu, J. Yang, W. Zhao et al., "Design and analysis of a new fault-tolerant permanent magnet vernier machine for electrical vehicles", *IEEE Trans. Magn.*, vol. 48, no. 11, pp. 4176-4179, 2012.
110. W. Zhao, C. Gu, Q. Chen et al., "Remedial phase-angle control of a five-phase fault-tolerant permanent-magnet vernier machine with short-circuit fault", *CES Trans. Electr. Mach. Syst.*, vol. 1, no. 1, pp. 83-88, 2017.
111. T. Zou, X. Han, D. Jiang et al., "Modeling and sensorless control of an advanced concentrated winding vernier PM machine", *Proc. IEEE Int. Conf. on Electrical Machines*, pp. 1112-1118, September 2016.
112. X. Han, T. Zou, D. Jiang et al., "Modeling and sensorless vector control for vernier PM machines", *Proc. IEEE Int. Power Electronics and Motion Control Conf. (ECCE Asia)*, pp. 3355-3360, May 2016.
113. X. Luo, S. Niu and W. N. Fu, "Design and sensorless control of a novel axial-flux permanent magnet machine for in-wheel applications", *IEEE Trans. Appl. Supercond.*, vol. 26, no. 7, 2017.
114. Z.S. Du and T.A. Lipo, "High torque density ferrite permanent magnet vernier motor analysis and design with demagnetization consideration", *Proc. IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 6082-6089, September 2015.